# RELATIVE MOTION

## RELATIVE MOTION

Motion is a combined property of the object under study and the observer. Motion is always relative, there is no such term like absolute motion or absolute rest. Motion is always defined with respect to an observer or reference frame.

**Reference frame :**

Reference frame is an axis system from which motion is observed. A clock is attached to measure time. Reference frame can be stationary or moving. There are two types of reference frame:

**(i) Inertial reference frame :** A frame of reference in which Newton's first law is valid is called as inertial reference frame.

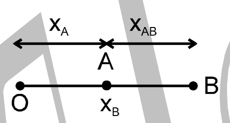
**(ii) Non-inertial reference frame :** A frame of reference in which Newton's first law is not valid is called as non-inertial reference frame.

**Note :** Earth is by definition a non-inertial reference frame because of its centripetal acceleration towards sun. But, for small practical applications earth is assumed stationary hence, it behaves as an inertial reference frame.

## RELATIVE VELOCITY

**Definition :** Relative velocity of a particle (object) A with respect to B is defined as the velocity with which A appears to move is B if considered to be at rest. In other words, it is the velocity with which A appears to move as seen by the B considering itself to be at rest.

**Relative motion along straight line -**




$$v_A = \frac{dx_A}{dt}, \qquad v_B = \frac{dx_B}{dt}$$

$$x_{BA} = x_B - x_A$$

$$v_{BA} = \frac{dx_B}{dt} - \frac{dx_A}{dt}$$

$$v_{BA} = v_B - v_A$$

$\Rightarrow \quad v_{AA} = v_A - v_A = 0$ (velocity of A with respect to A)

**Note :** velocity of an object w.r.t. itself is always zero.

**Ex.1** An object A is moving with 5 m/s and B is moving with 20 m/s in the same direction. (Positive x-axis)

(i) Find velocity of B with respect to A.

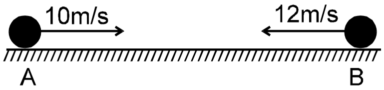
**Sol.** $v_B = 20\hat{i}$ m/s $\Rightarrow$ $v_A = 5\ \hat{i}$ m/s $\Rightarrow$ $v_B - v_A = 15\ \hat{i}$ m/s

(ii) Find velocity of A with respect to B

**Sol.** $v_B = 20\hat{i}$ m/s, $v_A = 15\hat{i}$ m/s $\Rightarrow$ $v_{AB} = v_A - v_B = -15\hat{i}$ m/s

**Note :** $\mathbf{v_{BA} = -v_{AB}}$

**Ex.2** Two objects A and B are moving towards each other with velocities 10 m/s and 12 m/s respectively a shown.




(i) Find out velocity of A with respect to B.

**Sol.** $v_{AB} = v_A - v_B = (10) - (-12)$ = 22 m/s towards right.

(ii) Find out velocity of B with respect to A

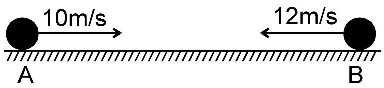
$v_{BA} = v_B - v_A = (-12) - (10)$ = –22 m/s towards left.

**Velocity of Approach**

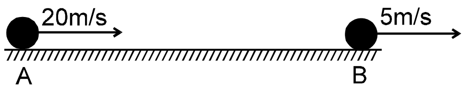
It is the rate at which a separation between two moving particles decreases.

If separation decreases velocity of approach is positive,

Velocity of approach = 22 m/s




Velocity of approach = 15 m/s

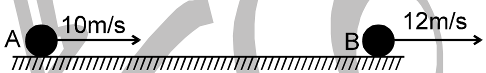



If separation increases, velocity of approach is negative. It is mainly called velocity of separation.

**Velocity of separation -**

It is the rate with which separation between two moving object increases.

Velocity of separation = 2 m/s




Velocity of separation = 15 m/s

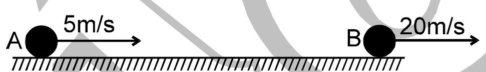
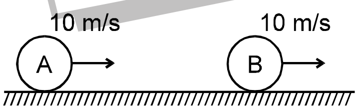



***Illustration :***

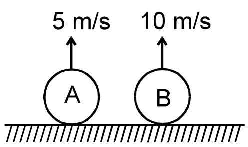
Two balls A and B are moving in the same direction with equal velocities, find out their relative velocity.




Velocity of A with respect to B $(\vec{v}_{AB}) = 0$

***Illustration :***

A and B are thrown vertically upward with velocity, 5 m/s and 10 m/s respectively (g = 10 m/s$^2$. Find separation between them after one second




**Sol.** $S_A = ut - \frac{1}{2} gt^2$ $= 5t - \frac{1}{2} \times 10 \times t^2$

$= 5 \times 1 - 5 \times 1^2$ $= 5 - 5$ $= 0$

$S_B = ut - \frac{1}{2} gt^2$ . $= 10 \times 1 - \frac{1}{2} \times 10 \times 1^2$

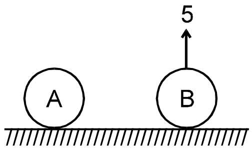
$= 10 - 5 \qquad\qquad = 5$

$\therefore S_B - S_A$ = separation = 5m.

**Alter :**

By relative $\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$

$= (-10) - (-10) = 0$



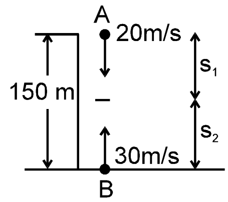

Also $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 10 - 5 \qquad = 5$ m/s

$\therefore \vec{s}_{BA}$ (in 1 sec) $= \vec{v}_{BA} \times t \qquad = 5 \times 1 \qquad = 5$ m

$\therefore$ Distance between A and B after 1 sec = 5 m.

***Illustration :***

A ball is thrown downwards with a speed of 20 m/s from top of a building 150 m high and simultaneously another ball is thrown vertically upwards with a speed of 30 m/s from the foot of the building. Find the time when both the balls will meet. (g = 10 m/s$^2$)




**Sol.** (I) $S_1 = 20\,t + 5\,t^2$

$+\ S_2 = 30\,t - 5\,t^2$

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$150 = 50\,t$

$\Rightarrow \quad t = 3$ s.

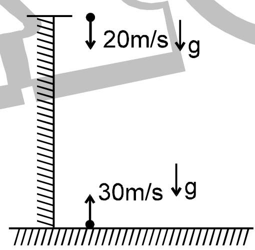
(II) Relative acceleration of both is zero since both have acceleration in downward direction

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

$= g - g = 0$

$\vec{v}_{BA} = 30 - (-20)$

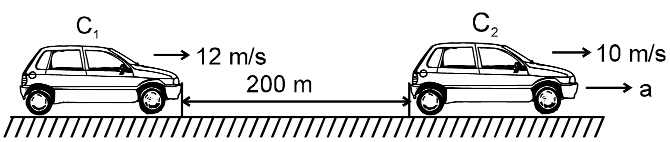
$= 50$




$s_{BA} = v_{BA} \times t$

$$t = \frac{s_{BA}}{v_{BA}} = \frac{150}{50} = 3 \text{ s}$$

**Ex.5** Two cars $C_1$ and $C_2$ moving in the same direction on a straight road with velocities 12 m/s and 10 m/s respectively. When the separation between the two was 200 m $C_2$ started accelerating to avoid collision. What is the minimum acceleration of car $C_2$ so that they don't collide.




**Sol.** By relative

$\vec{a}_{C_1C_2} = \vec{a}_{C_1} - \vec{a}_{C_2} \qquad = 0 - a \qquad = (-a)$



$\vec{v}_{C_1C_2} = \vec{v}_{C_1} - \vec{v}_{C_2} = 12 - 10 = 2$ m/s.

So by relativity we want the car to stop.

$\therefore\ v^2 - u^2 = 2as.$

$\Rightarrow\ 0 - 2^2 = -2 \times a \times 200 \qquad \Rightarrow\ a = \frac{1}{100}\ m/s^2$

$= 0.1\ m/s^2 \qquad = 1\ cm/s^2.$

$\therefore$ Minimum acceleration needed by car $C_2 = 1\ cm/s^2$

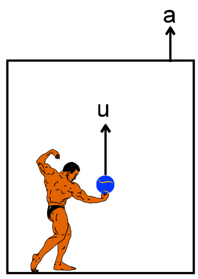
## RELATIVE MOTION IN LIFT

***Illustration :***

A lift is moving up with acceleration a. A person inside the lift throws the ball upwards with a velocity u relative to hand.

(a) What is the time of flight of the ball?

(b) What is the maximum height reached by the ball in the lift?




**Sol.** **(a)** $\vec{a}_{BL} = \vec{a}_B - \vec{a}_L = (g + a)$ downwards

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}_{BL}t^2 \qquad 0 = uT - \frac{1}{2}(g + a)T^2$

$\therefore \qquad T = \frac{2u}{(g+a)}$

**(b)** $v^2 - u^2 = 2as \qquad 0 - u^2 = -2(g + a)H$

$H = \frac{u^2}{2(g+a)}$

### Projectile motion in a lift moving with acceleration a upwards

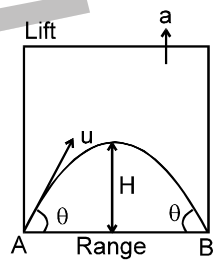
(1) Initial velocity = u

(2) Velocity at maximum height = u cos θ

(3) $T = \frac{2u\sin\theta}{g+a}$

(4) Maximum height (H) = $\frac{u^2\sin^2\theta}{2(g+a)}$

(5) Range = $\frac{u^2\sin 2\theta}{g+a}$




## RELATIVE MOTION IN TWO DIMENSION

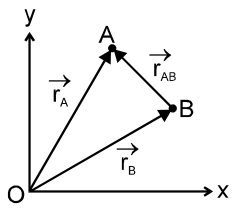
$\vec{r}_A$ = position of A with respect to O

$\vec{r}_B$ = position of B with respect to O

$\vec{r}_{AB}$ = position of A with respect to B.

$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$

$\therefore \qquad \frac{d(\vec{r}_{AB})}{dt} = \frac{d(\vec{r}_A)}{dt} - \frac{d(\vec{r}_B)}{dt}. \qquad \Rightarrow \qquad \vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

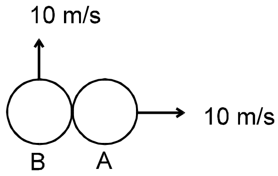


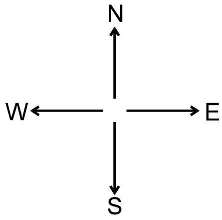
$$\frac{d(\vec{v}_{AB})}{dt} = \frac{d(\vec{v}_A)}{dt} - \frac{d(\vec{v}_B)}{dt} \qquad \Rightarrow \qquad \vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

**Note :** These formulae are not applicable for light.

***Illustration :***

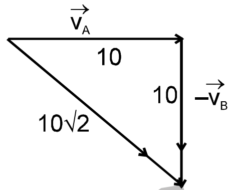
Object A and B has velocities 10 m/s. A is moving along East while B is moving towards North from the same point as shown. Find velocity of A relative to B ( $\vec{v}_{AB}$ )




**Sol.** $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$\therefore \ |\vec{v}_{AB}| = 10\sqrt{2}$




**Note :** $|\vec{v}_A - \vec{v}_B| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$

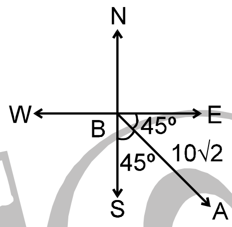
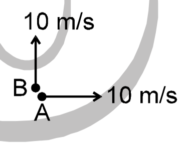



***Illustration :***

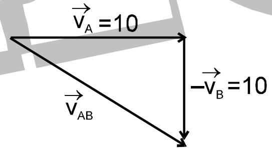
Two particles A and B are projected in air. A is thrown horizontally, B is thrown vertically up. What is the separation between them after 1 sec.



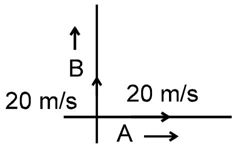

**Sol.** $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = 0$

$\therefore \quad \vec{v}_{AB} = \sqrt{10^2 + 10^2} = 10\sqrt{2}$

$\therefore \quad s_{AB} = v_{AB}t = (10\sqrt{2})\ t = 10\sqrt{2}$ m




Consider the situation, shown in figure




**Ex.23** (i) Find out velocity of B with respect to A

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A \qquad = 20\,\hat{j} - 20\,\hat{i}$$

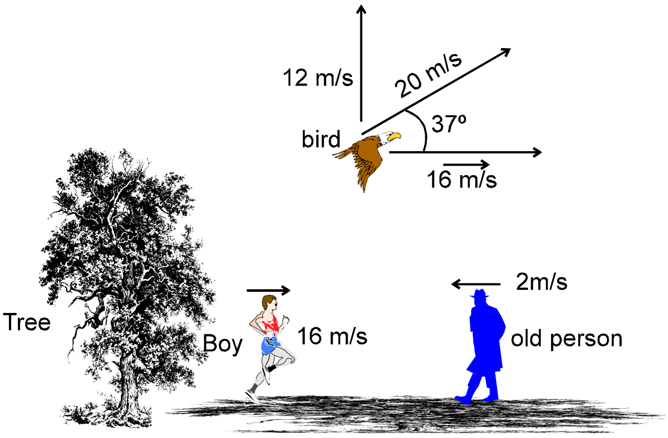
(ii) Find out velocity of A with respect to B

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \qquad = 20\,\hat{i} - 20\,\hat{j}$$

**Ex.24**






(1) Find out motion of tree, bird and old man as seen by boy.

(2) Find out motion of tree, bird, boy as seen by old man

(3) Find out motion of tree, boy and old man as seen by bird.

**Sol.** (1) With respect to boy :

$v_{tree}$ = 16 m/s (←)

$v_{bird}$ = 12 m/s (↑)

$v_{old\,man}$ = 18 m/s (←)

(2) With respect to old man :

$v_{Boy}$ = 18 m/s (→)

$v_{Tree}$ = 2 m/s (→)

$v_{Bird}$ = 18 m/s (→) and 12 m/s (↑)

(3) With respect to Bird :

$v_{Tree}$ = 12 m/s (↓) and 16 m/s (←)

$v_{old\,man}$ = 18 m/s (←) and 12 m/s (↓).

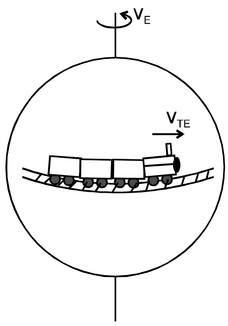
$v_{Boy}$ = 12 m/s (↓).

## MOTION OF A TRAIN MOVING ON EQUATOR :

If a train is moving at equator on the earth's surface with a velocity $v_{TE}$ relative to earth's surface and a point on the surface of earth with velocity $v_E$ relative to its centre, then

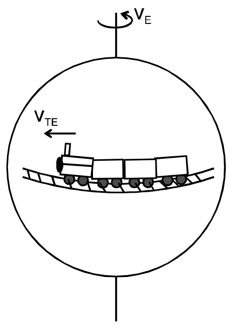
$\vec{v}_{TE} = \vec{v}_T - \vec{v}_E$ or $\vec{v}_T = \vec{v}_{TE} + \vec{v}_E$

So, if the train moves from west to east (the direction of motion of earth on its axis)




$v_T = v_{TE} + v_E$

and if the train moves from east to west (i.e. opposite to the motion of earth)




$v_T = v_{TE} - v_E$

**Relative Motion on a moving trains :**



If a boy is running with speed $\vec{v}_{BT}$ on a train moving with velocity $\vec{v}_T$ relative to ground, the speed of the boy relative to ground $\vec{v}_B$ will be given by:

$$\vec{v}_{BT} = \vec{v}_B - \vec{v}_T$$

or $$\vec{v}_B = \vec{v}_{BT} + \vec{v}_T$$

so, if the boy is running in the direction of train

$$v_B = u + v$$

and if the boy is running on the train in a direction opposite to the motion of train

$$v_B = u - v$$

## RELATIVE MOTION IN RIVER FLOW :

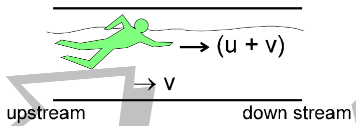
If a man can swim relative to water with velocity $\vec{v}_{mR}$ and water is following relative to ground with velocity $\vec{v}_R$, velocity of man relative to ground $\vec{v}_m$ will be given by :

$$\vec{v}_{mR} = \vec{v}_m - \vec{v}_R$$

or $$\vec{v}_m = \vec{v}_{mR} + \vec{v}_R$$

So, if the swimming is in the direction of flow of water,

$$v_m = v_{mR} + v_R$$




and if the swimming is opposite to the flow of water,

$$v_m = v_{mR} - v_R$$

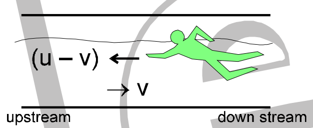



***Illustration :***

A swimmer capable of swimming with velocity v relative to water jumps in a flowing river having velocity u. The man swims a distance d down stream and returns back to the original position. Find out the time taken in complete motion.

**Sol.** $t = t_{down} + t_{up}$

$$= \frac{d}{v+u} + \frac{d}{v-u} = \frac{2dv}{v^2 - u^2}$$ **Ans.**

## CROSSING RIVER

A boat or man in a river always moves in the direction of resultant velocity of velocity of boat (or man) and velocity of river flow.

### 1. Shortest Time :

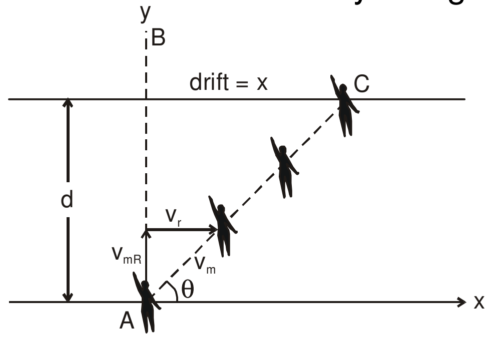
The person swims perpendicular to the river flow crossing a river : consider a river having flow velocity $\vec{v}_R$ and swimmer jump into the river from a point A, from one bank of the river, in a direction perpendicular to the direction of river current. Due to the flow velocity of river the swimmer is drifted along the river by a distance BC and the net velocity of the swimmer will be $\vec{v}_m$ along the direction AC.

If we find the components of velocity of swimmer along and perpendicular to the flow these are.

Velocity along the river, $v_x = v_R$.

Velocity perpendicular to the river, $v_f = v_{mR}$




The net speed is given by $v_m = \sqrt{v_{mR}^2 + v_R^2}$

at an angle of $\tan\theta = \dfrac{v_{mR}}{v_R}$ (down stream with the direction of flow).

Velocity of $v_y$ is used only in crossing the river, time taken to cross the river is $t = \dfrac{d}{v_y} = \dfrac{d}{v_{mR}}$.

Velocity $v_x$ is only used to drift the motion of the swimmer in the river, drift is along the river flow,

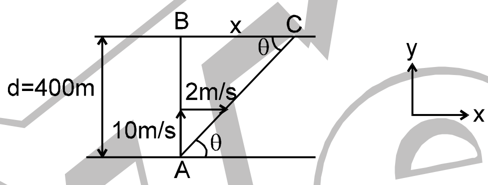
$$x = (v_x)\,(t) \qquad \text{or} \qquad x = v_R \frac{d}{v_{mR}}$$

***Illustration :***

A river 400 m wide is flowing at a rate of 2.0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.

(a) Find the time taken by the boat to reach the opposite bank.
(b) How far from the point directly opposite to the starting point does the boat reach the opposite bank.
(c) In what direction does the boat actually move.

**Sol.**




(a) time taken to cross the river

$$t = \frac{d}{v_y} = \frac{400\,m}{10\,m/s} = 40\text{ s} \qquad \textbf{Ans.}$$

(b) drift $(x) = (v_x)(t) = (2\text{m/s})\,(40\text{s}) = 80$ m **Ans.**

(c) Actual direction of boat,

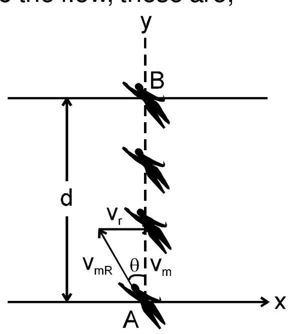
$$\theta = \tan^{-1}\left(\frac{10}{2}\right) = \tan^{-1} 5,\ \text{(downstream) with the river flow.}$$

## 2. SHORTEST PATH :

When the person crosses the river perpendicularly (along the shortest path). It should swim up stream making an angle θ with AB such that the resultant velocity $\vec{v}_m$, of man must be perpendicular to the flow of river along AB.

If we find the components of velocity of swimmer along and perpendicular to the flow, these are,

velocity along the river, $v_x = 0$




and velocity perpendicular to river $v_y = \sqrt{v_{mR}^2 - v_R^2}$

The net speed is given by $v_m = \sqrt{v_{mR}^2 - v_R^2}$

at an angle of 90º with the river direction.

velocity $v_y$ is used only to cross the river, therefore time to cross the river, $t = \frac{d}{v_y} = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}}$

and velocity $v_x$ is zero, therefore, in this case the drift (x) should be zero.

$x = 0$

or $v_x = v_R - v_{mR} \sin\theta = 0$

or $v_R = v_{mR} \sin\theta$

or $\theta = \sin^{-1}\left(\frac{v_R}{v_{mR}}\right)$

Hence, to cross the river perpendicular (along the shortest path) the man should swim at an angle of $\frac{\pi}{2} + \sin^{-1}\left(\frac{v_R}{v_{mR}}\right)$ upstream from the direction of river flow.

further, since $\sin\theta < 1$,

Swimmer can cross the river perpendicularly only when $v_{mR} > v_R$ ie

Practically it is not possible to reach at B if the river velocity ($v_R$) is too high.

***Illustration :***

A man can swim at the rate of 5 km/h in still water. A river 1 km wide flows at the rate of 3 km/h. The man wishes to swim across the river directly opposite to the starting point.

(a) Along what direction must the man swim?

(b) What should be his resultant velocity?

(c) How much time the would take to cross?

**Sol.** The velocity of man with respect to river $v_{mR}$= 5 km/hr, this is greater than the river flow velocity, therefore, he can cross the river directly (along the shortest path). The anlge of swim must be

$$\theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right) = 90^\circ + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right)$$

$$= 90^\circ + \sin^{-1}\left(\frac{3}{5}\right) = 90^\circ + 37^\circ = 127^\circ, \text{ with the river flow (upstream)}$$ **Ans.**

(b) Resultant velocity will be $v_m = \sqrt{v_{mR}^2 - v_R^2}$

$= \sqrt{5^2 - 3^2}$

$= 4$ km/hr

along the direction perpendicular to the river flow.

(c) time taken to cross the

$$t = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}} = \frac{1\,km}{4\,km/hr} = \frac{1}{4}\ h = 15\ min$$

**Ex.** The velocity of about in still water is 5 km/h it crosses 1 km wide river in 15 minutes along the shortest possible path. Determine the velocity of water in the river in km/h

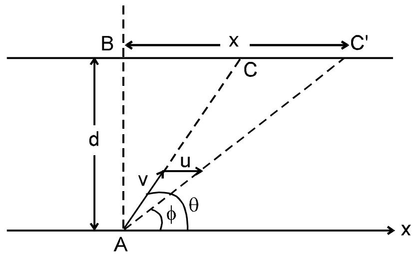
**Ans.** 3km/h

***Illustration :***

A man wishes to cross a river flowing with velocity u jumps at an anlge θ with the river flow. Find out the net velocity of the man with respect to ground if he can swim with speed v. Also find

How far from the point directly opposite to the starting point does the boat reach the opposite bank. in what direction does the boat actually move. If the width of the river is d.

**Sol.** Velocity of man = $v_M = \sqrt{u^2 + v^2 + 2vu\cos\theta}$ **Ans.**




$\tan\phi = \dfrac{v\sin\theta}{u + v\cos\theta}$ **Ans.**

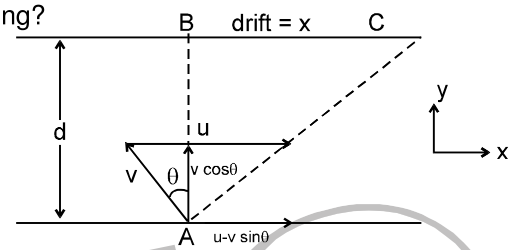
$(v \sin\theta)\, t = d \quad \Rightarrow \quad t = \dfrac{d}{v\sin\theta}$

$x = (u + v\cos\theta)\, t \quad = (u + v\cos\theta)\dfrac{d}{v\sin\theta}$ **Ans.**

***Illustration :***

A boat moves relative to water with a velocity which is n times less than the river flow velocity. At what anlge to the stream direction must the boat move to minimize drifting?

**Sol.** In this problem, one thing should be carefully meted that the velocity of boat is less than the river flow velocity. In such a case, boat cannot reach the point directly opposite to its starting point. i.e. drift can never be zero. Thus, to minimize the drift, boat starts at an angle θ from the normal direction up stream as shown.




Now, again if we find the components of velocity of boat along and perpendicular to the flow, these are,

velocity along the river, $v_x = u - v\sin\theta$.

and velocity perpendicular to the river, $v_y = v\cos\theta$.

time taken to cross the river is $t = \dfrac{d}{v_y} = \dfrac{d}{v\cos\theta}$.

In this time, drift $x = (v_x)t$

$= (u - v\sin\theta)\dfrac{d}{v\cos\theta}$

or $x = \dfrac{ud}{v}\sec\theta - d\tan\theta$

The drift x is minimum, when $\dfrac{dx}{d\theta} = 0$,

or $\left(\dfrac{ud}{v}\right)(\sec\theta \,.\, \tan\theta) - d\sec^2\theta = 0$

or $\dfrac{u}{v}\sin\theta = 1$

or $\sin\theta = \dfrac{v}{u} = \dfrac{1}{n} \quad (\text{as } v = \dfrac{u}{n})$

so, for minimum drift, the boat must move at an angle $\theta = \sin^{-1}\left(\dfrac{v}{u}\right)$ from normal direction or

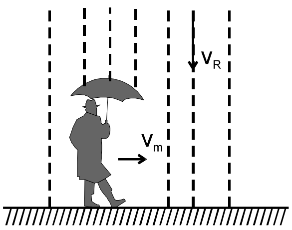
an angle $\dfrac{\pi}{2} + \sin^{-1}\left(\dfrac{v}{u}\right)$ from stream direction.

## RAIN PROBLEMS

If rain is falling vertically with a velocity $\vec{v}_R$ and on observer is moving horizontally with velocity $\vec{v}_m$, the velocity of rain relative to observer will be :

$$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m \qquad \text{or} \qquad v_{Rm} = \sqrt{v_R^2 + v_m^2}$$

and direction $\theta = \tan^{-1}\left(\dfrac{v_m}{v_R}\right)$ with the vertical as shown in figure.



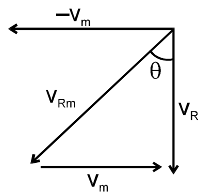
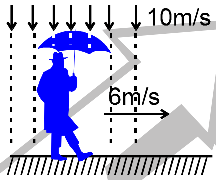



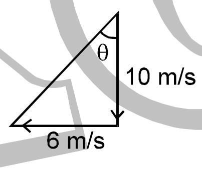

***Illustration :***

Rain is falling vertically and a man is moving with velocity 6 m/s. Find the angle with which umbrella should be hold by man to avoid getting wet.

**Sol.**




$\vec{v}_{rain} = -10\,\hat{j}$ $\qquad$ $\vec{v}_{man} = 6\,\hat{i}$



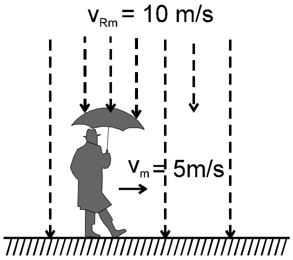

Velocity of rain with respect to man = $\vec{v}_{rm} = -10\,\hat{j} - 6\,\hat{i}$

$\tan\theta = \dfrac{6}{10}$ $\qquad$ $\theta = \tan^{-1}\left(\dfrac{3}{5}\right)$

Where θ is angle with vertical

***Illustration :***

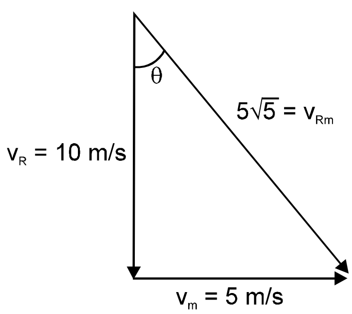
A man moving with 5m/s observes rain falling vertically at the rate of 10 m/s. Find the speed and direction of the rain with respect to ground.




$v_{RM} = 10$ m/s, $v_M = 5$ m/s

$\vec{v}_{RM} = \vec{v}_{Ru} - \vec{v}_M$

$\Rightarrow \qquad \vec{v}_{Ru} = \vec{v}_{RM} - \vec{v}_M$



$\Rightarrow \quad \vec{v}_R = 5\sqrt{5}$

$\tan\theta = \frac{1}{2}, \ \theta = \tan^{-1}\frac{1}{2}.$

***Illustration :***

A man standing, observes rain falling with velocity of 20 m/s at an angle of 30º with the vertical.

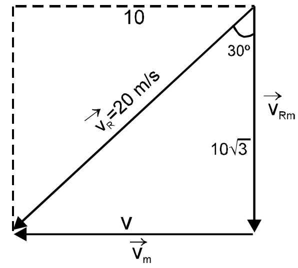
(1) Find out velocity of man so that rain appears to fall vertically.

(2) Find out velocity of man so that rain again appears to fall at 30º with the vertical.

**Sol.** (1) $\vec{v}_m = -v\,\hat{i}$ (let)

$\vec{v}_R = -10\,\hat{i} - 10\sqrt{3}\,\hat{j}$

$\vec{v}_{RM} = -(10 - v)\,\hat{i} - 10\sqrt{3}\,\hat{j}$




$\Rightarrow \quad -(10 - v) = 0$ (for vertical fall, horizontal component must be zero)

or $v = 10$ m/s **Ans.**

(2) $\vec{v}_R = -10\,\hat{i} - 10\sqrt{3}\,\hat{j}$

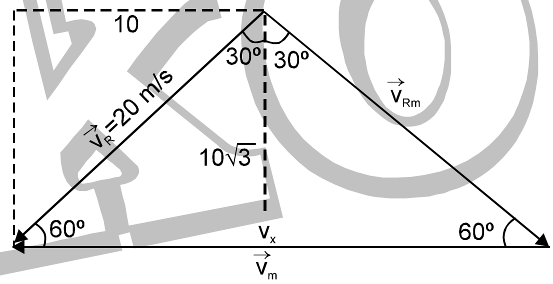
$\vec{v}_m = -v_x\,\hat{i}$

$\vec{v}_{RM} = -(10 - v_x)\,\hat{i} - 10\sqrt{3}\,\hat{j}$

Angle with the vertical = 30º

$\Rightarrow \quad \tan 30º = \frac{10 - v_x}{-10\sqrt{3}}$

$\Rightarrow \quad v_x = 20$ m/s




## WIND AIRPLANE

This is very similar to boat river flow problems the only difference is that boat is replaced by also plane and river is replaced by wind.

Thus,

velocity of aeroplane with respect to wind

$\vec{v}_{aw} = \vec{v}_a - \vec{v}_w$

or $\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$

where, $\vec{v}_a$ = absolute velocity of aeroplane

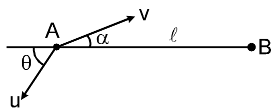
and, $\vec{v}_w$ = velocity of wind.

***Illustration :***

An aeroplane flies along a straight path A to B and returns back again. The distance between A and B is $\ell$

and the aeroplane maintains the constant speed v. There is a steady wind with a speed u at an angle θ with line AB. Determine the expression for the total time of the trip.

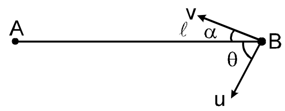
**Sol.** A to B :




Velocity of plane along AB = v cosα – ucosθ ,
and for no-drift from line

AB : v sinα = usinθ $\Rightarrow$ $\sin\alpha = \dfrac{u\sin\theta}{v}$

time taken from A to B : $t_{AB} = \dfrac{\ell}{v\cos\alpha - u\cos\theta}$




B to A :
velocity of plane along BA = vcosα + u cosθ
and for no drift from line AB : vsinα = usinθ

$\Rightarrow$ $\sin\alpha = \dfrac{u\sin\theta}{v}$

time taken from B to A : $t_{BA} = \dfrac{\ell}{v\cos\alpha + u\cos\theta}$

total time taken = $t_{AB} + t_{BA}$

$$= \frac{\ell}{v\cos\alpha + u\cos\theta} + \frac{\ell}{v\cos\alpha + u\cos\theta}$$

$$= \frac{2v\ell\cos\alpha}{v^2\cos^2\alpha + u^2\cos^2\theta} = \frac{2v\ell\sqrt{1 - \dfrac{u^2\sin^2\theta}{v^2}}}{v^2 - u^2}.$$

**Ex.** Find the time an aeroplane having velocity v, take to fly around a square with side a and the wind blowing at a velocity u, in the two cases,

(a) if the direction of wind is along one side of the square,
(b) If the direction of wind is along one of the diagonals of the square

**Ans.** (a) $\dfrac{2a}{v^2 - u^2}\left(v + \sqrt{v^2 - u^2}\right)$ (b) $2\sqrt{2}\,a\left(\dfrac{\sqrt{2v^2 - u^2}}{v^2 - u^2}\right)$.

**Condition to collide or to reach at the same point**

When the relative velocity of one particle w.r.t. to other particle is directed towards each other then they will collide. (If there is a zero relative acceleration).

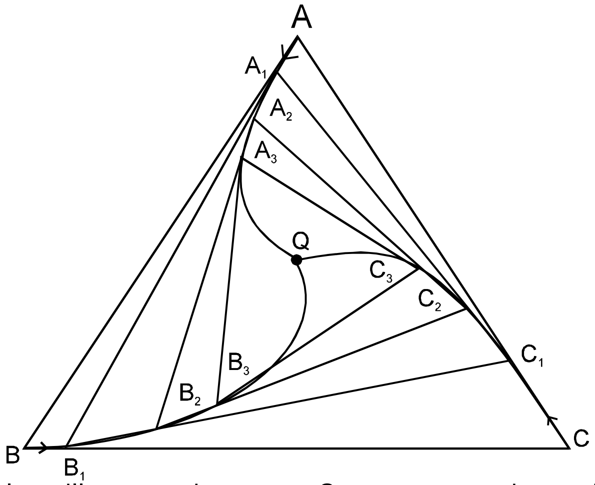
***Illustration :***

There are particles A, B and C are situated at the vertices of an equilateral triangle ABC of side a at t = 0. Each of the particles moves with constant speed v. A always has its velocity along AB, B along BC and C along CA. At what time will the particle meet each other?

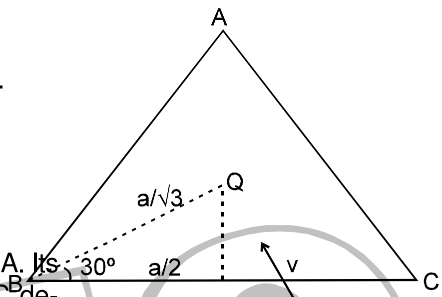
**Sol.** The motion of the particles is roughly sketched in figure. By symmetry they will meet at the centroid O of the triangle. At any instant the particles will from an equilateral triangle ABC with the same






Centroid O. All the particles will meet at the centre. Concentrate on the motion of any one particle, say B. At any instant its velocity makes angle 30º with BO.
The component of this velocity along BO is v cos 30º. This component is the rate of decrease of the distance BO. Initially.

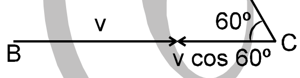



$$BO = \frac{a/2}{\cos 30^\circ} = \frac{a}{\sqrt{3}} = \text{displacement of each particle.}$$

Therefore, the time taken for BO to become zero

$$= \frac{d/\sqrt{3}}{v\cos 30^\circ} = \frac{2d}{\sqrt{3}v \times \sqrt{3}} = \frac{2d}{3v}.$$

**Alternative :** Velocity of B is v along BC. The velocity of C is along CA. Its component along BC is v cos 60º = v/2. Thus, the separation BC decreases at the rate of approach velocity.




$\therefore$


$$\therefore \text{ approach velocity} = v + \frac{v}{2} = \frac{3v}{2}$$

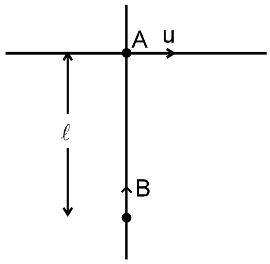
Since, the rate of approach is constant, the time taken in reducing the separation BC from a to zero is

$$t = \frac{a}{\frac{3v}{2}} = \frac{2a}{3v}$$

**Q.** Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

**Ans.** 2 a/v.

**Q.** 'A' moves with constant velocity u along then 'x' axis. B always has velocity towards A. After how much time will B meet A if B moves with constant speed v. What distance will be travelled by A and B.




**Ans.** distance travelled by A = $\frac{v^2\ell}{v^2 - u^2}$,

distance travelled by B = $\frac{uv\ell}{v^2 - u^2}$

***Illustration :***

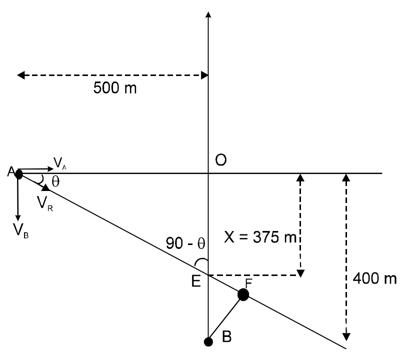
Two cars A and B are moving west to east and south to north respectively along crossroads. A moves with a speed of 72 $kmh^{-1}$ and is 500 m away from point of intersection of cross roads and B moves with a speed of 54 $kmh^{-1}$ and is 400 m away from point of intersection of cross roads. Find the shortest distance between them ?

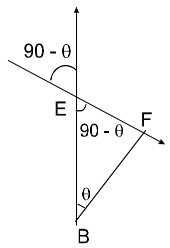
**Sol.**

**Method – I (Using the concept of relative velocity)**






In this method we watch the velocity of A w.r.t. B. To do this we plot the resultant velocity $V_r$. Since the accelerations of both the bodies is zero, so the relative acceleration between them is also zero. Hence the relative velocity will remain constant. So the path of A with respect to B will be straight line and along the direction of relative velocity of A with respect to B. The shortest distance between A & B is when A is at point F (i.e. when we drop a perpendicular from B on the line of motion of A with respect to B).

From figure

$$\tan\theta = \frac{V_B}{V_A} = \frac{15}{20} = \frac{3}{4} \quad \text{........................(i)}$$

This θ is the angle made by the resultant velocity vector with the x-axis.

Also we know that from figure

$$OE = \frac{x}{500} = \frac{3}{4} \quad \text{..............................(ii)}$$

From equation (i) & (ii) we get

x = 375 m

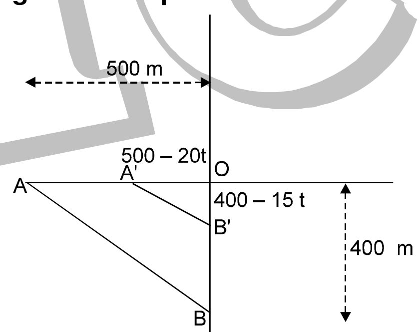
∴ EB = OB – OE = 400 – 375 = 25 m

But the shortest distance is BF.

From magnified figure we see that BF = EB cosθ $= 25 \times \frac{4}{5}$

∴ BF = 20 m

**Method II (Using the concept of maxima – minima)**




A & B be are the initial positions and A',B' be the final positions after time t.

B is moving with a speed of 15 m/sec so it will travel a distance of BB' = 15t during time t.

A is moving with a speed of 20 m/sec so it will travel a distance of AA' = 20t during time t.

So

OA' =500 – 20 t

OB' = 400 – 15 t

∴ $A'B'^2 = OA'^2 + OB'^2 = (500 - 20t)^2 + (400 - 15t)^2$ .................(i)

For A'B' to be minimum $A'B'^2$ should also be minimum

$$\therefore \quad \frac{d(A'B'^2)}{dt} = \frac{d(400-15t)^2 + (500-20t)^2}{dt} = 0$$

= 2(400 – 15t) (–15) + 2(500 – 20t) (–20) = 0

= – 1200 + 45t = 2000 – 80 t

∴ 125 t = 3200

$\therefore$ $t = \frac{128}{5}$ s. Hence A and B will be closest after $\frac{128}{5}$ s.

Now $\frac{d^2A'B'}{dt^2}$ comes out to be positive hence it is a minima.

On substituting the value of t in equation (i) we get

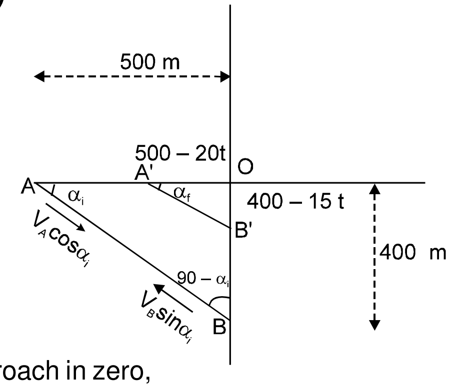
$\therefore$ $A'B'^2 = \left(400 - 15\times\frac{128}{5}\right)^2 + \left(500 - 20.\frac{128}{5}\right)^2$

$= \sqrt{16^2 + (-12)^2} = 20$ m

$\therefore$ Minimum distance A'B' = 20 m.

**Method III (Using the concept of relative velocity of approach)**

After time t let us plot the components of velocity of A & B in the direction along AB. When the distance between the two is minimum, the relative velocity of approach is zero.




$\therefore$ $V_A \cos\alpha_f + V_B \sin\alpha_f = 0$

(where $\alpha_f$ is the angle made by the line A'B' with the x-axis)

$20 \cos\alpha_f = -15 \sin\alpha_f$

$\therefore$ $\tan\alpha_f = -\frac{20}{15} = -\frac{4}{3}$

Here do not confuse this angle with the angle $\theta$ in method (I) because that $\theta$ is the angle made by the resultant with x-axis.

Here $\alpha_f$ is the angle made with x-axis when velocity of approach in zero,

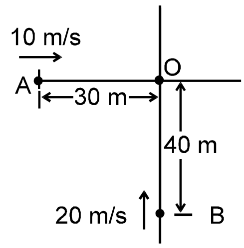
$\therefore$ $\frac{400-15t}{500-20t} = -\frac{4}{3}$

$\therefore$ $t = \frac{128}{5}$ So, OB' = 16 m and OA' = – 12m

$A'B' = \sqrt{16^2 + (-12)^2} = 20$ m

**Qus.** Two ships are 10 km apart on a line running south to north. The one farther north is steaming west at 20 km $h^{-1}$. The other is steaming north at 20 km $h^{-1}$. What is their distance of closest approach ? How long do they take to reach it ?

**Ans.** $5\sqrt{2}$ km/h ; 1/4 h = 15 min consider the situation shown in figure for the two particle A and B.




**Qus.** (1) Will the two particle will collide

(2) Find out shortest distance between two particles

**Ans.** (1) The particles will not collide

(2) $4\sqrt{5}$ m.

**Note : Muzzle Velocity is the velocity of bullet with respect to the gun i.e. it is Relative Velocity.**